PROJECT

TEMPORAL COMPLEXITY IN BIOLOGICALLY PLAUSIBLE FORMAL NEURAL NETWORKS

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In the nervous system, intrinsic properties of neurones and synapses on one hand, architectural features of the network on the other hand contribute to the neuronal dynamics.

Models of neural networks based on neurones of Hodgkin-Huxley (HH) type exhibit rich dynamical properties reproducing quite well some neural activities observed in different structures of the nervous system. In such models however, a systematic exploration of the dynamical role of the neuronal intrinsic properties and the features of the networks can't be performed because of the huge number of involved parameters. Moreover, the number of differential equations is such that time integration limits the number of neurones in a network. The simplified neuronal models of Integrate&Fire (IF) type reduce the number of parameters providing informative simulations with large networks (see [1], for instance). But these models do not allow a distinction between the intrinsic properties of the neurones and of the synapses, and the dynamical impact of the network architecture features. In a network of discrete time units as the MCP ones, the biological plausibility is far lower than in the previously mentioned continuous time models, HH or IF.

The networks of the binary MCP neurones allow the study of *dynamical properties induced by the connectivity parameters only*, these parameters being the signs, the weights and the delays of the synaptic connections. In this context we have shown that a recurrent network of MCP neurones (200 units), with equal populations of excitatory and inhibitory neurones, synchronously updated, reveal rich dynamical behaviours. In many cases, it exhibits complex spatio-temporal activities based on oscillations of the number of active neurones. The groups of active neurones change from one oscillation to another one, realizing patterns characterised by 'distributed synchrony' which can be characterized using a complexity index, the NED (Normalised Euclidean Distance) [2]. Such patterns are good candidates to be spatio-temporal codes : they remind the neural activity recorded experimentally in the locust olfactory system. Indeed, in response to an applied stimulus, some neurones reveal a non stimulus-specific oscillation, figuring an "internal clock", co-emergent with a stimulus-specific spatio-temporal pattern of these neurones [3].

In order to evaluate the contribution of intrinsic properties of neurones and synapses, we implemented the architecture of the MCP network into a network of *continuous time neurones*. As a first step, we chose the Izhikevich model (IZ) for its biological plausibility combined with its computational efficiency [4]. With this type of network it is possible to display the temporal evolution of the membrane potential of each unit, including its spiking activity. Under some conditions, which imply a temporal organisation and static synaptic saturations, we have shown that the IZ network behaves exactly as the MCP one [5]. As a second step, we can remove the temporal constraints related to the architecture so that the dynamics of the network is essentially driven by the neuronal intrinsic properties. The further step consists on the introduction of dynamic synapses, which may play a major role in setting clocks in the system.

Step 1: Clocks in networks of discrete and continuous time formal neurones

ABSTRACT

For the study of neuronal dynamics in the central nervous system, we use neural network simulations aiming at distinguishing the contributions of the *network architecture* from the *neurone intrinsic properties*. Indeed, the specific features of these two levels, respectively *recurrence* and *non-linearity,* are at the origine of complex dynamical behaviours In addition, each level brings its own temporal constraints.
A rich repertoire of dynamical behaviours emerge in McCulloch&Pitts (MCP) networks, as a result of interactions of the network architecture and the non-linear simple units. Such networks are good tools in order to explore the impact of the network connection parameters on its dynamics. In order to show that these studies are biologically relevant, we go further toward their biological plausibility by replacing the discrete time MCP neurons by continuous time neurons, for instance the Izhikevich model (IZ). There are many differences between these two types of neurons, however once defined some appropriate updating rules for the IZ model, *any network architecture* implemented in both models lead them to *exhibit exactly the same spatio-temporal activity*. The present step of the project leads us to analyze the different time scales involved in the two models, the synchrony of the updating, and the emergence of an "internal clock" due to the architecture.

KEY WORDS
Modeling, Binary units, Izhikevich model, Neural networks, Complex dynamics.

A same dynamical behaviour for networks of two different neuronal models

1.1 MCP network

In a simulation of a network of MCP neurones, we can consider *two time scales*.

(I) The first one is related to the neurone update processing. Each update processing takes a computation time in order to compute the value 0 or 1 for the state of each of the binary units of the network, according to the balance of its active afferent synapses. The computational time step Δt is the update time step. When a neurone is updated at a given time, it means that, at this time, the state of the network is available for this unit and it is entirely defined by the synaptic balance. An MCP neuron is called a 'discrete time' neuron.
If *all* the units of the network are updated within every Δt of the simulation, the dynamics *is synchronous*; if only some of the units are updated, the dynamics is asynchronous. There is only one rule for synchrony, while several rules may be defined for asynchrony. At the end of each time step, we get the state of the

network. Let us call t, this instant: the state of the network at t is given by the binary vector $\vec{s}(t)$ whose dimension is the size of the network. The next update instant is defined as ($t+1$), the unit (1) figuring the time step Δt. Therefore, an MCP network has an implicit, *intrinsic* clock; with a synchronous update, at each beat of the clock the state of each neuron can be either 0 or 1, *whatever its state at the previous beat*;. This clock defines a temporal grid on which the dynamics of the network settles down.

(II) When the network activity is characterized by a periodic or quasi-periodic global behaviour of the neurones it induces *a second time scale.* An eigen-frequency dominates the dynamics, eliciting an eigen-time τ, multiple of Δt, basis of an *emergent internal clock*. This internal clock depends *only* on the network architecture, which includes all the connectivity parameters, as synaptic signs, weights and delays. The spatio-temporal complexity feature of the neural activities around the emergent internal clock is also a product of the architecture. This complexity can be characterized by a scalar descriptor, the 'Normalized Euclidean Distance' (NED) [2], which measures the temporal distribution of the neuronal activities along the successive periods of the internal clock oscillations. With this descriptor, we can study the relationship between some parameters of the network architecture and the network behaviour. The possibility to study such structure-function relationships is one of the major advantages offered by the MCP networks, another one being the rapidity of getting simulations.

Now, what is the degree of relevance of such studies in the framework of questions about the dynamics of *biological* neurons? We are looking for the conditions which allow a network of continuous time neurons, more biologically plausible IZ neurones for instance, to exhibit neuronal activities *entirely driven by the network architecture*. In other words, which temporal constraints may lead an IZ network to exhibit a neural dynamics *strictly identical* to the one of a *same* network (i.e. a network equivalent from the architectural point of view) of MCP neurons?

1.2 IZ network

Two major properties distinguish an IZ continuous time neurone from an MCP one. The first one lies on the fact that in an IZ neuron, between the silent state and the emission of a spike, there is not the symmetry which characterises the states 0 and 1 of the binary MCP neuron; in addition, there is also an asymmetry of the excitatory or inhibitory synaptic influences. The second property lies on the existence of a latency before the emission of a spike, when an excitatory input is strong enough to elicit it in an IZ neuron. *This latency depends on the intensity* of the excitation and on the presence or not of some inhibition.

The IZ neuron model we consider here is described in [5]. It captures some properties of realistic HH type conductance-based models with biological plausibility and has an interesting simplicity of hardware implementation. In this model, the potential of each unit is given by a two-dimensional system of ordinary differential equations.

(I) Here a first time scale is directly related to numerical computation. It is based on the *integration time step* related to the differential equations modeling the dynamics of the membrane potential of the IZ neuron. This integration time is set with respect to the time constants appearing in these differential equations; it defines the degree of precision of the membrane potential computation: the smaller this integration time step, the higher the precision. The computational time depends on it: the smaller this integration step, the slower the simulation. Therefore, when defining the integration time step, it is necessary to take into account the computational power of the machine, in order to get simulations within reasonable time duration. As it was described in the MCP model for Δt, it is possible to attribute a unitary value to the elementary computation time, δt, during which the computation of the value of the membrane potential of each neuron in the network is performed. At each instant t ending an elementary time step δt, we can associate a potential state of the network by defining a real vector $\vec{v}(t)$, each coordinate of which corresponds to the instantaneous value of the membrane potential of one of the neurons, value belonging to the interval [-80mV, +30mV] typically. A binary vector representing the state of the neurons at t, $\vec{s}(t)$, can be easily deduced from $\vec{v}(t)$, by giving value 1 to the neurons whose membrane potential is equal to 30mV and 0 otherwise. At time $(t+1)$, $\vec{v}(t+1)$ and $\vec{s}(t+1)$ can be computed as well, but if the ith coordinate of $\vec{s}(t)$, i.e. the state of neuron i, is 1, *it can't be* 1 *again at* $(t+1)$, but must be 0. This rule is true for the state of neuron i until t', such as $(t'-t)$ *is greater than the absolute refractory period*, T_r, of the neuron model. This rule about the possible states of the IZ neurons along successive instants of this computational clock differs essentially from the rule driving the neuronal binary states along successive instants of the MCP network intrinsic clock. Therefore, if we want to compare the neural dynamics in both types of neural networks, *we can't rely on equivalence between these two clocks, in other words: between δt and Δt.* Moreover, the integration time step at the basis of δt is typically one or two order of magnitude smaller than the refractory period T_r, which could be compared to what represents Δt, the time step between two successive updates in the MCP network.

(II) Because of the very different intrinsic computational clocks involved in IZ and MCP simulations, we can not expect to get the same activity in the two types of network. For instance, when taking the connection matrix, the delay matrix and the input vector from a MCP network showing a "distributed synchrony" to implement a network of IZ neurones, this IZ network exhibits a *very different* spatio-temporal pattern: there is no distributed synchrony, but a quite regular activity of all the neurones excited by the input.

Thus, in order to get the same pattern in both networks, we have to define an *explicit temporal grid* for the update rule in the IZ network. This grid is an *extrinsic clock which structures the dynamics*. The time step Δt_{IZ} associated with this new clock is typically of the order of magnitude of T_r.

On one hand, the time step Δt_{IZ} is defined according to some constraints: (i) Δt_{IZ} is larger than T_r in order to insure a possible attribution of the value 1 for any neuron at two successive beats of the new clock, and, moreover, (ii) Δt_{IZ} must be large enough in order to erase any 'memory' of the state of the neuron between two beats, which means that the membrane potential of every neuron that has emitted a spike at a given beat has been reset to its resting value before the next beat.

On the other hand, in the MCP network, the information about the synaptic state of the network is available at each beat of the intrinsic clock, this means that a synapse with unit delay is active at time t as soon as its pre-synaptic neuron was in state 1 at time $(t-1)$. Accordingly, in the IZ network, it is the time step, Δt_{IZ}, of the extrinsic clock which figures the smallest synaptic transmission time. If different synaptic delays are taken into account, they are multiple of Δt_{IZ}.

For each neurone, its synaptic balance is the algebraic sum of external (input) and internal (network) synaptic weights: it is introduced as an applied current I in one of the two combined differential equations computing the membrane potential V of the IZ neurons. In order to get exactly the *same* dynamics in both models when they are implemented with the *same* architecture, we apply to I the rule of the MCP. It means that I is a Heaviside function of the synaptic balance at each "update" time which will be defined now. According to a pulse train function f defined by pulses of width t_w, occurring at period Δt_{IZ}, at each pulse I is given either a saturation value I_{sat}, if the synaptic balance is positive or a nil value. Thus I realizes a kind of digital/analogic converter, on the basis of the temporal grid f. I_{sat} and t_w are set in order to get a perfect bijection between the binary values of I { 0, I_{sat} }, and the absence or presence of *a single spike* in the post-synaptic neuron. In addition, when the period Δt_{IZ}, is defined as described above, we get the bijection at each "update" time, i.e at each beat of the clock set by f. For instance, $I_{sat}=6$, $t_w=5\,\delta t$, and $\Delta t_{IZ}=100\delta t$ are convenient when the differential equations defining the IZ neurones are the following:

By setting I to a saturation value for all neurones whose synaptic balance is positive at a given beat of this clock, we get quite a good synchronization of their spikes. Then, at a given update time, the internal part of the synaptic balances is computed from the neurones that have emitted spikes during fixed time windows

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around the previous update times, phase locked to the pulse train, with a width exceeding t_w, considering their contribution according to the delay matrix.

(III) With the extrinsic clock defined above, the spatio-temporal pattern exhibited by the spikes in the IZ network *is exactly the same* as the activity map of the binary MCP neurons. Therefore, it is also possible to get a quasi-periodic activity in a network of IZ neurons when its architecture induces it in the network of MCP ones, with the corresponding *emergence of an internal clock* whose characteristic time is τ, same multiple of Δt_{IF}, as it is of Δt.

Conclusion

We have shown that the neural dynamics due to the architectural properties of a neural network can be complex enough in order to get the co-emergence of both an internal clock and a possible spatio-temporal code: this is what can be proven when such an architecture is implemented in an MCP network, where the units are deprived of individual dynamical properties. We describe here how the introduction of an appropriate temporal grid (an extrinsic clock) in simulations of a IZ network allows this one to exhibit the *same* dynamics as the one of an MCP network, provided that *both networks are build according to the same architecture*. Therefore, it is possible to consider the extrinsic clock imposed to the IZ network as a tool which allows to *isolate the dynamical impact of the network properties* from that of the individual dynamical properties of the continuous time IZ neurons. This also validates the pertinence of MCP networks as auxiliary models to study the neural dynamics in the biological systems.

Conversely, now that we know how to define temporal constraints in the network of IZ neurons in order to get exactly the activity of their equivalent MCP neurons, we can imagine to relax these constraints and to define, at the level of the MCP neurons this time, new rules in order to keep a dynamical equivalence between these two types of network. Typically, the new rules may act at the level of the asynchrony of the neuronal update, which could be a way of introducing elements of continuous time neuron dynamics in an MCP neural network, in addition to its architectural properties. Then, we aim at introducing, combined with the neural network, a network of synapses, as transmitting objects modeling the dynamical behaviour of biological synapses.

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